White Paper:
Establishing and Investigating Foundation Zones of Influence

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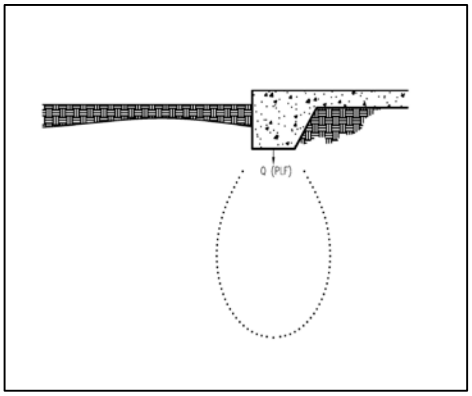
Introduction



Determining a reasonable depth for the zone of influence is required during the design phase of construction projects to estimate the amount of settlement a structure may undergo due to the proposed change in loading to the subgrade. This method commonly known as the Boussinesq [ˈbü·siˈnesk] method is widely used throughout the geotechnical industry. In addition to estimating the settlement of structures supported on shallow foundations this method is also used to determine what influence a foundation may have on neighboring structures.

Because this method is well-established for use in estimating the zone of soil affected or influenced by the structure it seems reasonable to conclude that this method could be used to determine the zone of soil acting to support or influence the structure. In other words, the Boussinesq method is a pre-construction design tool that can be reconfigured for use as a post-construction analysis tool. As an analysis tool this modified Boussinesq equation could then be used to determine the impact of post-construction events or activities acting on the soils within the zone of influence.

This paper serves to introduce and explain the Boussinesq method used for design. This paper then breaks down the basic equation and reconfigures it plugging in known values so as to solve for and ultimately determine the zone of soil acting to support or influence the structure. These findings are then presented graphically and in table format for use in on-going analysis efforts.

This paper concludes with a discussion of future efforts intended to be conducted so as to further refine the Boussinesq method used for analysis. Specifically, the basis for each of the equations assumptions will be explored and actual in situ testing performed so as to develop tabularized values for use.

Background

The most critical area to investigate with respect to the geotechnical aspect of a post-construction structural analysis is the influence zone, or the zone of soil that is significantly influenced by an applied surface load (i.e., the load applied to the soil by the foundation of a residential structure). For all practical purposes, the influence zone can be defined as the stressed zone beneath a foundation which is responsible for the settlement of a structure. The depth of significant influence can generally be understood as the depth to which applied stresses are significantly felt in the soil. Typically, strains in the soil are considered insignificant once the stresses have attenuated to a value of 10 to 15% of those at the surface. Karl Terzaghi observed that direct stresses in the soil are considered of negligible magnitude when they are less than 20% of the intensity of the applied stress from structural loading.

For the purposes of this paper, we will consider the depth of significant influence to mean that depth below the foundation of a structure where 15% of the applied stress due to structural loading is “felt” by the soil.

Boussinesq Method

One of the most widely used formulas to compute the vertical stress increase in a soil mass due to an applied stress was published by Joseph Boussinesq. In 1885, Joseph Boussinesq, a French mathematician, first solved the problem of determining stresses below the surface of a semi-infinite solid using the linearized theory of elasticity.

In order to use the linearized theory of elasticity to determine vertical stress increases in a soil mass, a number of assumptions must be made. First is the assumption that the soil mass we are concerned with will behave like a linear elastic material. This may not seem like a valid assumption given the fact that, in general, soil response is neither linear nor elastic. However, at stress levels less than or equal to approximately one-third of the soil’s ultimate strength, the behavior of a soil is at least approximately linear. This can be illustrated graphically by plotting the relationship between the axial stress and axial strain of a soil. And, since foundations are usually designed with a safety factor of three or more, the vertical stress increase within the soil under normal loading conditions will almost always fall within this range.

A second set of assumptions involves the soil mass being semi-infinite, homogeneous and isotropic. A semi-infinite mass is one that is bounded on one side by a horizontal surface, in our case the surface of the earth, and is infinite in all other directions. A soil is considered to be homogeneous if there are identical elastic properties at every point in the mass in identical directions. A soil is considered to be isotropic if there are identical elastic properties throughout the mass and in every direction through any point of it. While there is no such thing as a truly homogenous soil stratum with identical properties throughout found in nature, theory and experience have shown that the shape of the pressure distribution on any horizontal section below the loaded surface is more or less independent of the physical properties of the loaded subgrade. Further, experience has also shown that the increase in vertical pressure can be computed with sufficient accuracy on the assumption that the subsoil is perfectly elastic and homogeneous. Therefore, computing the change in vertical stress within a given soil mass based on the linearized theory of elasticity, despite the necessary assumptions, is a valid exercise.

Westergaard

Another method of calculating the vertical stress increases in a soil mass, which utilizes the linearized theory of elasticity, was developed by Westergaard. Westergaard's analysis has been shown to more closely represent the elastic conditions of a stratified soil mass by assuming that the soil is reinforced by thin, nonyielding horizontal sheets of negligible thickness. Westergaard's formula may be applied when the soil contains alternating layers of stiff and soft material. Experience has shown that the vertical stress increases below a stiff soil stratum will be less than what is indicated by Boussinesq's equation thereby reducing the depth of the influence zone. Therefore, when the near-surface soils contain a stiff material layer such as a very dense sand or stiff clay, the influence zone can be more accurately determined using the formula developed by Westergaard. However, a reasonable and conservative estimate of the size of the influence zone is provided by the Boussinesq equation based on its conservative nature.

Design Considerations

Having briefly discussed the linearized theory of elasticity as it applies to the subgrade we must then consider how the load bearing components of a structure affect the foundation geometry and ultimately the zone of influence. Typical substructure components of a residential building provide continuous support for the load-bearing walls and consist of either a monolithic footing constructed 12 inches wide or a stem-wall footing constructed 16 or 24 inches wide. Foundations such as these are characterized as "infinitely long" or "continuous" strip footings since the length of the footing is greater than 10 times the width.

Typically, factors are applied to the structural loads used for foundation design in order to achieve a certain factor of safety. Additionally, it is standard industry practice to apply a safety factor of three to the ultimate bearing capacity of the subgrade to account for any variability in the bearing strata. Therefore, if the foundation has been designed to meet all requirements of the applicable building codes and design plans, the footing width will incorporate the appropriate safety factors. And, since the footing width ultimately governs the extent to which the soil is significantly influenced by the structure, it is unnecessary to apply an additional factor of safety to the size of the influence zone from an analysis perspective.

Analysis

Given these foundation considerations, Boussinesq's equation can be used to determine the depth at which the stress increase in the soil mass reaches 15% of the intensity of the load at the surface. Also, if our point of interest (i.e. the depth of the influence zone) is greater than twice the width of the footing, the strip load generated by the foundation may be treated as a line load (force per unit length). However, the line load equation provides more conservative values (a higher stress increase) than the strip load equation in soils shallower than twice the width of the foundation.

Boussinesq's equation for determining the vertical stress increase ($\Delta\sigma_v$) in the soil due to a line load is shown below.

$$\sigma_z = \Delta\sigma_v = \frac{2Qz^3}{\pi(x^2 + z^2)^2}$$

The Bousinessq equation for a line load requires three variables to determine the vertical stress increase at a given point in the soil due to an applied surface load. Those variables represent the force per unit length of foundation (Q), the depth below the foundation where the stress increase is to be determined (z), and the horizontal distance from the line load (x).

Since the greatest increase in vertical stress will be induced below the center line of the foundation, the variable (x) in the Boussinesq equation will be equal to zero. With (x) set to zero and simplifying (z) we are left with a simple mathematical calculation.

$$\sigma_z = \Delta\sigma_v = \frac{2Q}{\pi(z)}$$

The following examples illustrate the steps involved in calculating the depth of the influence zone.

Example 1

First, let us consider a certain wood-framed residential structure whose load bearing walls are supported by a continuous 12-inch wide monolithic footing that applies a pressure of 800 psf to the subgrade. The first step in our analysis is to solve Bousinessq's line load equation for the variable (z). This process is simplified by the fact that the (x) term is equal to zero for reasons described above. Solving the equation for (z) gives us:

$$z = \frac{2Q}{\pi \Delta \sigma_v}$$

The next step is to determine the magnitude of the line load responsible for generating 800 psf through the 12-inch wide continuous footing. This is accomplished as follows:

$$800\, psf * 1.0\, ft = 800\, plf$$

This number is represented by the variable (Q) in the line load equation. Next is to calculate the magnitude of the vertical stress increase that represents 15% of the applied stress at the surface due to structural loading. This is determined simply by multiplying the bearing pressure by 15% as shown below:

$$800\, psf * 15\% = 120\, psf$$

Thus, we have determined all variables necessary to obtain the depth of the influence zone. Plugging in values for the known quantities gives us:

$$z = \frac{(2)(800\, plf)}{(3.14)(120\, psf)} = 4.25\, ft$$

Based on the method and assumptions described above, the depth of the influence zone, or the depth at which applied stresses are significantly felt in the soil, can be considered to extend to a depth of 4.25 feet below the bottom of a 12-inch wide foundation.

Example 2

To illustrate how the width of the foundation affects the depth of the influence zone, we will consider another example. This time, let us consider a two-story masonry structure with a wood-framed second story whose load bearing walls are supported by a continuous 24-inch wide strip footing that applies a pressure of 1,500 psf to the subgrade.

Following the same procedure outlined above, after having solved the line load equation for (z) we determine the magnitude of the line load responsible for generating 1,500 psf through a 24-inch wide continuous footing.

$$1{,}500\, psf * 2.0\, ft = 3{,}000\, plf$$

Next we determine the magnitude of the vertical stress increase that represents 15% of the applied stress at the surface due to structural loading as shown:

$$1{,}500\, psf * 15\% = 225\, psf$$

Again, having determined values for all the necessary variables we are left with the following calculation:

$$z = \frac{(2)(3{,}000\, plf)}{(3.14)(225\, psf)} = 8.49\, ft$$

Zone of Influence Depth

The examples above serve to illustrate the fact that the influence zone of a typical residential foundation can be considered to fall between 4.25 and 8.49 feet below the bottom of the foundation. The depth and lateral extent of the zone of influence is dependent on the width of the foundation and can be represented graphically by tracing a line through all points below the ground surface where the increases in vertical stress are equal.

For the purpose of our discussion herein, a line connecting all points where the magnitude of the vertical stress increase equals 15% of the total applied stress is the one of practical significance. The line representing such a stress contour resembles the form of a bulb extending vertically from the bottom of the foundation to a depth between 4.25 and 8.49 feet.

Zone of Influence Width

After the depth of the pressure bulb is determined, our next task is to determine the lateral extent of such a stress contour. Thus, we will have determined the maximum extent, both vertically and horizontally, to which the soils are influenced by the foundation of a structure.

In order to determine the lateral extent of the influence zone, Boussinesq's line load equation for determining vertical stress increases must be solved for (x). This is accomplished as follows:

$$\Delta\sigma_v = \frac{2Qz^3}{\pi(x^2 + z^2)^2}$$

$$\Delta\sigma_v(x^2 + z^2)^2 = \frac{2Qz^3}{\pi}$$

$$(x^2 + z^2)^2 = \frac{2Qz^3}{\pi\Delta\sigma_v}$$

$$(x^2 + z^2) = \left(\frac{2Qz^3}{\pi\Delta\sigma_v}\right)^{1/2}$$

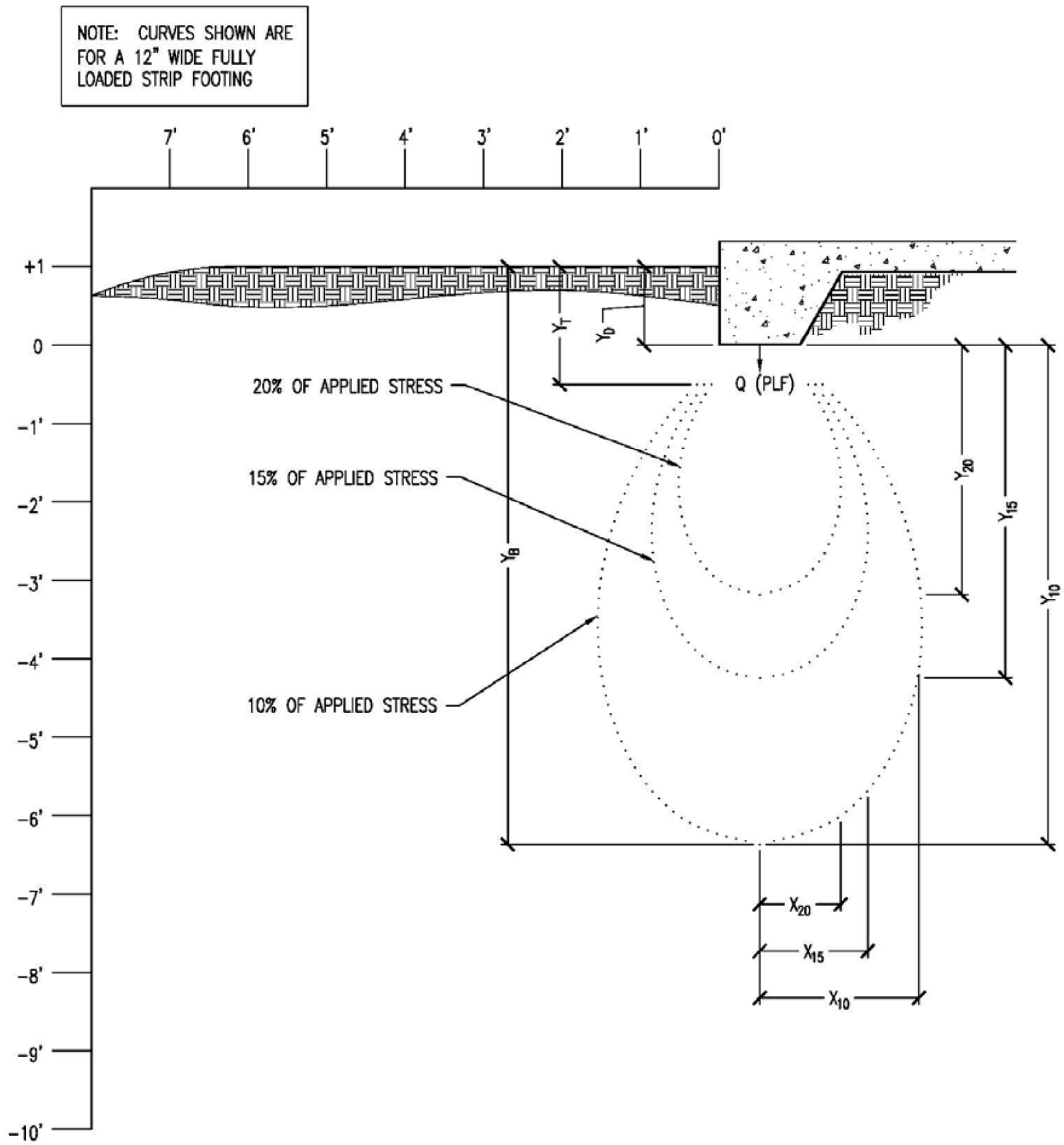
$$x^2 = \left(\frac{2Qz^3}{\pi\Delta\sigma_v}\right)^{1/2} - z^2$$

$$x = \left[\left(\frac{2Qz^3}{\pi\Delta\sigma_v}\right)^{1/2} - z^2\right]^{1/2}$$

This equation leaves us with known values for (Q) and ($\Delta\sigma_v$), and unknown values for (x) and (z).

Since the equation above contains two unknowns, namely the depth (z) and horizontal distance (x), iterations must be performed by varying the depth (z) until the maximum value of (x) is obtained. This value of (x) will be the maximum horizontal distance away from the center line of the foundation where the soil is significantly influenced by structural loading.

This process is simplified by the use of a spreadsheet. Generally, a depth equal to roughly half of the maximum depth of the influence zone is a good place to begin the iterations. Performing these steps for a 24-inch wide strip footing indicates a maximum distance of 2.75 feet away from the center line at a depth of 4.75 feet below the base of the foundation.



ZONE OF INFLUENCE BASED ON BOUSSINESQ METHOD AT 10%, 15% AND 20%

Conclusion

Because the Boussinesq method is well-established for use in estimating the zone of soil affected or influenced by structures, it has been shown that this method can also be used to determine the zone of soil acting to support or influence the structure. In other words, the Boussinesq method is a pre-construction design tool that can be reconfigured for use as a post-construction analysis tool. As an analysis tool this modified Boussinesq equation is then effective at determining the impact of post-construction events or activities acting on the soils within the zone of influence.

Recommendations

It is recommended that the basis for each of the equations assumptions namely that the soil mass behaves like a linear elastic material and the soil mass acting as a semi-infinite, homogeneous and isotropic material be explored and actual in situ testing performed so as to develop tabularized values for use.

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